Learning Trajectories of Early Math

Julie Sarama & Douglas H. Clements
University of Denver

Vision

• 4- and 5-year-olds
• Puzzle
• Cory puts 4 triangles together to make squares

Cory makes a new shape: A unit of units!

Another boy sees the square structure, but builds the wrong square

Finishing, Cory shows adult, who asks: “How many triangles did you use?”

Cory counts: “24”
“24 what?”
“Triangles.”
“How many squares do you have?”
Puts 4 fingers on triangles in each new unit and counts each square: “6!”
Learning Trajectories

- Mathematics of children— representations and thinking of children as it developments naturally
- Activities matched to children’s development in each topic
- Therefore:
  - All within developmental capacities of children
  - Provide a natural “building block” to the next level
  - Provides mathematical building blocks for school success

Why Might You Care?

- How might you use LTs?
  - Keeping track
  - Checking up
  - Finding out
  - Perhaps most powerful, underused teaching strategy: Formative assessment
The Quality of Children’s Experiences

- 16 teachers of 6, 7-year-olds, considered above average, in the U.K. Dedicated, conscientious.
- More than 1/2 of tasks were mismatched
  - misdiagnosis
  - failures in task design
- High attainers were underestimated—41%
  - never even recognized that they underestimated
  - in no case was a task considered too easy
  - 80% more practice tasks than intended
  - children were "cheerful and industrious" and didn't mind doing the same old work

The Quality of Children’s Experiences

- Low attainers overestimated—44%
- Only 1/9 of time moved to a lower level
- Most of the time, just moved to next set of tasks

Formative Assessment

- Increases achievement more than most interventions
- Teachers’ assessments "have effect sizes from .4 to .7 standard deviations, larger than most effects of instructional programs, which are considered impressive with a .25 effect size” (Lorrie Shepard)
Learning Trajectories:
3 Parts

1. Goal
2. Developmental Progression
3. Instructional Activities

Learning Trajectory for Counting

1st: Goal: Accurate, confident object counting

2nd: Developmental Progression…

- **Precounter**  Says number words, not sequence: "one, two, four…"
- **Chanter**  Says in sequence but may run together
- **Reciter** Verbal counting to 5, then 10
• **Corresponder** Counts correctly using 1-1 correspondence, at least up to 5 objects in a line.
Learning Trajectory for Counting

- **Corresponder**: Counts correctly using 1-1 correspondence, at least up to 5 objects in a line

- **Counter (Small Numbers)**: Counts 1-5 objects in a line meaningfully (i.e., employ the cardinal rule)

- **Producer (Small Numbers)**: Counts out a collection up to 5

- **Counter (10)**: Counts collections up to 10

- **Counter and Producer (10+)**: and keeps track of unorganized collections
Learning Trajectory for Counting

- Counter from N
- Counter On Using Patterns
- Counter On Keeping Track
- Counter Forward and Back

Poll #1
What level of thinking?

1. Chanter
2. Reciter
3. Corresponder
4. Counter (Small Numbers)
5. Producer (Small Numbers)

Poll #2
What level of thinking?

1. Chanter
2. Reciter
3. Corresponder
4. Counter (Small Numbers)
5. Producer (Small Numbers)

Instructional Activities: 3rd Part of Learning Trajectories

Small Numbers and Counting

- Finger plays:
  - When I was one...
    - When I was one, I was so small. (hold up 1 finger)
    - I could not speak a word at all. (shake head)
    - When I was two, I learned to talk. (hold up 2 fingers)
    - I learned to sing, I learned to walk. (point to mouth and feet)
    - When I was three, I grew and grew. (hold up 3 fingers)
    - Now I am four and so are you! (hold up 4 fingers)
  - Later: Five Little Monkeys, etc.

Count and Move
Counting Circle

Books Limited

- 68% include numbers less than or equal to 10
- Only 12% present the number 0 in comparison to 90 percent of the books that presented the number 1.
- Less than ½ present 3 representations (numeral, number word, and quantity)
1 One dot can make a sun

41

42

43

44
Simon Says

Path Game…and Beyond

Get Goldilocks Home

Start

Finish
What Number Now?

Counting Games

Road Race: Connecting Representations

- Count the dots and move that number of jumps
- Connecting different representations of number!
Road Race Shape Counting - Another Variation

- Count the sides of a shape and move that number of jumps
- Connecting new concepts of number

Poll #3

What level of thinking is this teaching/practicing?
1. Chanter
2. Reciter
3. Corresponder
4. Counter (Small Numbers)
5. Producer (Small Numbers)

Space Race Number Choice

- Choose the “better” of two numbers
- Comparing but also reasoning: Which is better in this case?
A Trajectory for Composing Geometric Shapes
Substitution Composer

- Finds different ways to fill a frame, emphasizing substitution relationships.

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Create A Scene

Results: Child Assessment

- F(1, 32) = 40.52, p = .000+
- T Scores:
  - 50 Mean
  - 10 SD

TRIAD II: Large-Scale Evaluation

**TRIAD = Technology-enhanced, Research-based, Instruction, Assessment, and professional Development**

Building Blocks In the News

Using the Learning Trajectories

It takes time. A teacher talks about interviewing a child for report cards:

“She was able to do verbal counting to 8, and then when she slowed down, she could get to 11. So I said, “Can you make me a group of 6?” And so she did. So then I added, I did 12, I think. She couldn’t do it.

Then I noted that, so now I’m thinking in the trajectories, I think she’s a “Counter (Small Numbers),” right? She’s on her way to being a “Counter (10).” She’s in between the two. So that’s what I was thinking of as I did this.”

—Pat, 2004

Web Sites and Contact

Connect4Learning.com
TRIADscaleup.org

“If we teach today as we taught yesterday, we rob our children of tomorrow.” –John Dewey