Learning Trajectories of Early Math

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Learning Trajectories

- Mathematics of children—representations and thinking of children as it develops naturally
- Activities matched to children's development in each topic
- Therefore:
  - All within developmental capacities of children
  - Provide a natural “building block” to the next level
  - Provides mathematical building blocks for school success, because research based on typical higher-income children

Learning Trajectories: 3 Parts

1. Goal
2. Developmental Progression
3. Instructional Activities

Developmental Path:
The activities are designed to help children move along the developmental path. For example, children who can count out loud are engaged in activities that help them learn to count a collection of objects and understand that the last number named is the number in the collection.

Instructional Path:
For each topic, such as “counting,” research and practice have helped identify levels of thinking through which children move as they develop these skills. For example, they move from a level at which they can only count out loud, to counting objects successively, to even more sophisticated counting strategies.
Scientific Approach to Learning Trajectories weaves the 3 parts together

Big Ideas

Research: Teachers who study and focus on big ideas more successful
What Might Be Missed

- Learning Trajectories at the Core of the Common Core

Learning Trajectory for Counting

1st Goal: Accurate, confident object counting
Learning Trajectory for Counting

1st: Goal: Accurate, confident object counting

2nd: Developmental Progression…

• Precounter Says number words, not sequence: "one, two, four…”.
• Chanter Says in sequence but may run together
• Reciter Verbal counting to 5, then 10

• Corresponder Counts correctly using 1-1 correspondence, at least up to 5 objects in a line.
• Counter (Small Numbers) Counts 1-5 objects in a line meaningfully (i.e., employ the cardinal rule)
- **Producer (Small Numbers)** Counts out a collection up to 5

- **Counter (10)** Counts collections up to 10

- **Counter and Producer (10+)** and keeps track of unorganized collections

- **Counter from N**

- **Counter On Using Patterns**

- **Counter On Keeping Track**

- **Counter Forward and Back**
Instructional Activities: 3rd Part of Learning Trajectories

Small Numbers and Counting

- Finger plays:
  - One, two, buckle…
  - When I was one…

  When I was one, I was so small, (hold up 1 finger) I could not speak a word at all. (shake head)
  When I was two, I learned to talk. (hold up 2 fingers) I learned to sing, I learned to walk. (point to mouth and feet)
  When I was three, I grew and grew. (hold up 3 fingers) Now I am four and so are you! (hold up 4 fingers)

- Later: Five Little Monkeys, etc.

Count and Move

Small Group Record Sheet
Building Blocks Math - PreK Assessment

<table>
<thead>
<tr>
<th>Trajectory Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Collection</td>
<td>Can name collection of 5 objs</td>
</tr>
<tr>
<td>Maker of Small Collections</td>
<td>Can make obj up to 4</td>
</tr>
<tr>
<td>Counter (Small Numbers)</td>
<td>Can count up to 5 codes</td>
</tr>
<tr>
<td>Perceptual Subitizer to 4</td>
<td>Can subitize obj up to 5</td>
</tr>
<tr>
<td>Perceptual Subitizer to 5</td>
<td>Can subitize up to 5</td>
</tr>
</tbody>
</table>

Week: 2
Activity: Find and Make Groups

<table>
<thead>
<tr>
<th>Child’s Name</th>
<th>Finds groups to:</th>
<th>Strategies/ Trajectory Level</th>
<th>Comments</th>
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</thead>
<tbody>
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</tbody>
</table>
Activities for multiple goals

- What goals on number table?
  - Make and imagine small collections items *nonverbally*
  - Count by ones to 10
  - Know the last counting word tells “how many”
  - Count out (produce) a collection
  - Subitize (quickly “see” and label with a number)
  - Identify whether collections are the “same” number or which is “more” visually

Counting Games

- Count the dots and move that number of jumps
- Connecting different representations of number!
Road Race Shape Counting - Another Variation

- Count the sides of a shape and move that number of jumps
- Connecting new concepts of number

Space Race Number Choice

- Choose the “better” of two numbers
- Comparing but also reasoning: Which is better in this case?

Arithmetic Sequence

Encourage counting on from numeral
Add numerals
Addition “choice” game
Two-digit addition

A Trajectory for Composing Geometric Shapes
Substitution Composer

- Finds different ways to fill a frame, emphasizing substitution relationships.

We Lay Groundwork Early…

- “First, I drew a triangle…
- then I had a trapezoid…
- then a parallelogram…
- And when I was having hexagons,
- I still had 10 triangles!”

Create A Scene
Using the Learning Trajectories

It takes time. A teacher talks about interviewing a child for report cards:

“She was able to do verbal counting to 8, and then when she slowed down, she could get to 11. So I said, “Can you make me a group of 6?” And so she did. So then I added, I did 12, I think. She couldn’t do it.

Then I noted that, so now I’m thinking in the trajectories, I think she’s a “Counter (Small Numbers),” right? She’s on her way to being a “Counter (10).” She’s in between the two. So that’s what I was thinking of as I did this.”

—Pat, 2004

Web Sites (and article download)

TRIADscaleup.org
BuildingBlocksMath.org

“If we teach today as we taught yesterday, we rob our children of tomorrow.” –John Dewey